Astrophys Space Sci
DOI 10.1007/s10509-013-1542-y

ORIGINAL ARTICLE

On the existence of the relative equilibria of a rigid body in the J_2 problem

Yue Wang · Shijie Xu · Liang Tang

Received: 28 April 2013 / Accepted: 17 June 2013

Abstract The motion of a point mass in the J_2 problem has been generalized to that of a rigid body in a J_2 gravity field for new high-precision applications in the celestial mechanics and astrodynamics. Unlike the original J_2 problem, the gravitational orbit-rotation coupling of the rigid body is considered in the generalized problem. The existence and properties of both the classical and non-classical relative equilibria of the rigid body are investigated in more details in the present paper based on our previous results. We nondimensionalize the system by the characteristic time and length to make the study more general. Through the study, it is found that the classical relative equilibria can always exist in the real physical situation. Numerical results suggest that the non-classical relative equilibria only can exist in the case of a negative J_2, i.e., the central body is elongated; they cannot exist in the case of a positive J_2 when the central body is oblate. In the case of a negative J_2, the effect of the orbit-rotation coupling of the rigid body on the existence of the non-classical relative equilibria can be positive or negative, which depends on the values of J_2 and the angular velocity Ω_e. The bifurcation from the classical relative equilibria, at which the non-classical relative equilibria appear, has been shown with different parameters of the system. Our results here have given more details of the relative equilibria than our previous paper, in which the existence conditions of the relative equilibria are derived and primarily studied. Our results have also extended the previous results on the relative equilibria of a rigid body in a central gravity field by taking into account the oblateness of the central body.

Keywords J_2 problem · Rigid body · Gravitational orbit-rotation coupling · Classical relative equilibria · Non-classical relative equilibria

Y. Wang (✉) · S. Xu
Department of Guidance, Navigation and Control, School of Astronautics, Beijing University of Aeronautics and Astronautics, Room B1024, New Main Building, 100191 Beijing, China
e-mail: ywang@sa.buaa.edu.cn

S. Xu
e-mail: starsjxu@yahoo.com.cn

L. Tang
Science and Technology on Space Intelligent Control Laboratory, 100190 Beijing, China
e-mail: tl614@hotmail.com

L. Tang
Beijing Institute of Control Engineering, 100190 Beijing, China

1 Introduction

The J_2 problem, also known as the main problem of artificial satellite theory, is one of the most important problems in both celestial mechanics and astrodynamics, as the most significant non-spherical mass distribution of the central celestial body, i.e., the zonal harmonic J_2, is taken into account (Broucke 1994; Wang 2013a). In the J_2 problem, the motion of a point mass in a gravity field truncated on the zonal harmonic J_2 is studied. The J_2 problem has broad applications in the orbital dynamics and orbital design of spacecraft, such as the design of the sun synchronization orbits and the J_2 invariant relative orbits in the spacecraft formations (Xu et al. 2012). This classical problem has been studied in many works, such as Broucke (1994) and the literatures cited therein.

However, neither natural nor artificial celestial bodies are point masses or have spherical mass distributions. A practical generalization of the point mass model is the assumption that the body considered is perfectly rigid that is precise

Published online: 06 July 2013