

Effective potential energy in Størmer's problem for an inclined rotating magnetic dipole

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Abstract We discuss the dynamics of a charged nonrelativistic particle in electromagnetic field of a rotating magnetized celestial body. The equations of motion of the particle are obtained and some particular solutions are found. Effective potential energy is defined on the base of the first constant of motion. Regions accessible and inaccessible for a charged particle motion are studied and depicted for different values of a constant of motion.

Keywords Størmer's problem · Magnetic dipole · Electromagnetic field · Inclined rotator · Charge · Equation of motion

1 Introduction

The field of magnetic dipole and motion of charged particles in this field have large practical significance for astrophysics. Magnetic field of many planets and stars can be thought of as a dipole field in good approximation. Stationary case, when the magnetic moment of a celestial body coincides with the rotation axis, is well studied. Particularly, motion of a charged particle in the Earth magnetic field is studied in detail. Solution of equation of motion for charged particle in a dipolar magnetic field gives trapping regions for particles, which have energy of limited range. These regions of planets are named radiation belts (Alfven 1950; Holmes-Siedle and Adams 2002). The first theoretical studies on the properties of the trajectories of a charged particle

in the dipolar field were done by Størmer (1907), DeVogelaere (1958) and Dragt (1965). Rather complicated mathematical methods were developed in order to calculate the trajectory of charged particle in the dipole magnetic field (see for example Lanzano 1968). Solution of the Størmer's problem was extended to magnetic field that is a superposition of the field of a dipole and a uniform magnetic field. Allowed and forbidden regions of the motion of charged particles in such field was studied by Katsiaris and Psillakis (1986).

There are also well known celestial bodies, which direction of magnetic moment differs from the direction of axis of rotation. In this case electric field is induced in the neighborhood of the body by magnetic field. The neutron stars and pulsars are examples of such bodies. The first model of electric field which is generated in the neighborhood of a neutron star was developed by Deutsch (1955). Some other models were suggested and studied by several authors (Michel 1991). All these models are based on suggestion that the neutron star is a conducting body.

Conducting body rotating in its own magnetic field, generates a potential difference between the poles and equator. This potential difference leads to development of a co-rotating magnetosphere. By assuming that the magnetic field is dipolar, and unaffected by the trapped particles in the magnetosphere, and that the field dipole axis is parallel to the rotation axis, Goldreich and Julian (1969) determined many of the properties of the magnetosphere. Electromagnetic field and magnetosphere of the oblique rotator and conducting body have been studied by many authors. See for example Cohen and Rosenblum (1972), Kaburaki (1981), Cohen and Kearney (1980).

Electromagnetic field of conducting body differs essentially from the pure dipole field. But there are also celestial bodies, which consist of non-conducting matter and their axis of rotation is inclined with respect to the magnetic field

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