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ORIGINAL ARTICLE

Periodic motions in the spatial Chermnykh restricted three-body problem

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Abstract Three-dimensional motions in the Chermnykh restricted three-body problem are studied. Specifically, families of three-dimensional periodic orbits are determined through bifurcations of the family of straight-line periodic oscillations of the problem which exists for equal masses of the primaries. These rectilinear oscillations are perpendicular to the plane of the primaries and give rise to an infinite number of families consisting entirely of periodic orbits which belong to the three-dimensional space except their respective one-dimensional bifurcations as well as their planar terminations. Many of the computed branch families are continued in all mass range that they exist.

Keywords 3D Periodic orbits · R3BP · Chermnykh problem · Sitnikov motion · Bifurcations

1 Introduction

The restricted three-body problem (R3BP) is the most celebrated problem in Celestial Mechanics for both theoretical and practical point of view (see e.g. Szebehely 1967; Arnold 1987; Gómez et al. 2001). The dynamical structures of this simplified model-problem make it relevant for numerous applications in problems of orbital dynamics such as the determination of orbits of natural bodies in our Solar system, i.e. planets, asteroids or comets, or the detection of low energy transit trajectories for space mission designs (see e.g., Markellos 1974; Murison 1989; Howell and Kakoi 2006; Dutt and Sharma 2011). Modifications of the restricted problem have been proposed by several researchers in order to make it a more relevant model for specific applications, such as the photogravitational R3BP where the additional force of the radiation pressure is included in the model (Schuerman 1980; Simmons et al. 1985; Elipe and Lara 1997; Das et al. 2009) or the R3BP which takes into account the oblateness effect of a primary (Subba Rao and Sharma 1997; Oberti and Vienne 2003; Beevi and Sharma 2012).

Here, we consider the Chermnykh restricted three-body problem, a modified R3BP which simulates the motion of a massless body in the orbital plane of a dumb-bell which is rotating with constant angular velocity ω around the center of mass of the system (Chermnykh 1987). This problem is a generalization of the Euler's problem of two fixed gravitational centers ($\omega = 0$) and the R3BP ($\omega = 1$) and may have applications to Dynamical Astronomy as well as in Chemistry, for example to the quantization of the molecular ion of Hydrogen H_2^+ (for details, see Goździewski and Maciejewski 1998). The Chermnykh problem has also been studied by Perdios and Ragos (2004) and Papadakis (2005). Recently, modifications of the Chermnykh problem have been proposed such as the model in which the gravitational influence from a belt around the central binary is included in the model (Jiang and Yeh 2006) or the model in which the radiation and the oblateness of the primaries are taking into account (Kushvah 2008, 2009).

In this respect, we study motions in the three-dimensional space of the Chermnykh problem and in particular we deal with the computation of 3D periodic orbits. To this end, we follow a different way for their determination than that of the classical approach, i.e. we do not compute them from bifur-

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