Astrophys Space Sci (2013) 345:73–83
DOI 10.1007/s10509-013-1375-8

ORIGINAL ARTICLE

Periodic orbits and bifurcations in the Sitnikov four-body problem when all primaries are oblate

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Received: 1 July 2012 / Accepted: 23 January 2013 / Published online: 8 February 2013

Abstract We study the motions of an infinitesimal mass in the Sitnikov four-body problem in which three equal oblate spheroids (called primaries) symmetrical in all respect, are placed at the vertices of an equilateral triangle. These primaries are moving in circular orbits around their common center of mass. The fourth infinitesimal mass is moving along a line perpendicular to the plane of motion of the primaries and passing through the center of mass of the primaries. A relation between the oblateness-parameter 'A' and the increased sides 'ε' of the equilateral triangle during the motion is established. We confine our attention to one particular value of oblateness-parameter $A = 0.003$. Only one stability region and 12 critical periodic orbits are found from which new three-dimensional families of symmetric periodic orbits bifurcate. 3-D families of symmetric periodic orbits, bifurcating from the 12 corresponding critical periodic orbits are determined. For $A = 0.005$, observation shows that the stability region is wider than for $A = 0.003$.

Keywords Sitnikov problem · Oblate spheroid · Oblateness-parameter · Stability · Critical periodic orbits · Bifurcation

1 Introduction

Sitnikov problem is the simplest unsolved case of general n-body problem. But in astronomy it can be used as the first approximation in many real situations. This has led to a particular emphasis on periodic orbits which play an important role in the problem and on their stability. In the planer restricted three-body problem, the horizontal branches of a family of symmetric simple-periodic orbits, are families of symmetric multiple-periodic orbits of the second generation (Poincarè deuxieme genre) which bifurcating from the generating family of simple orbits. Markellos (1974, 1977) has found many of the horizontal branches of the family 'f' for the Sun-Jupiter value $\mu = 0.00095$.

When the restricted problem is extended in three-dimensional (3-D), it is found that the planer families also have vertical branches which bifurcate from the generating families at vertical self resonant orbits. Originally this type of problem was introduced by Pavanini (1907). Sitnikov himself (1960) first proved oscillatory solution of this problem, so the name *Sitnikov problem.* The problem has also been studied by many others, e.g. Hènon (1973) has established the criteria for 3-D periodic orbits bifurcating from the vertical critical orbits ($a_v = 1$). Markellos (1977) has studied bifurcations of plane with 3-D asymmetric periodic orbits in the restricted three-body problem. Zagouras and Markellos (1977) have determined the axisymmetric periodic orbits of the restricted three-body problem. Markellos (1978) has investigated bifurcation of straight-line oscillation; Robin and Markellos (1980) have developed whole mechanism by which vertical branches consisting of symmetric, three-dimensional periodic orbits, bifurcating from the families of plane orbits, at vertical self resonant orbit can be obtain. They also have investigated the relationship between orbital multiplicity, symmetry classes and type of mirror configurations. Moser (1973) has developed a chaotic orbit (such orbit was already mentioned by Poincarè in 1892) in the vicinity of escaping solution based on the work of Conley (1969). Using the perturbation theory; Hagel and Lhotka (2005), has derived the solution of linearized equation of motion.

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