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LETTER

Comments on "Two exact solutions to the general relativistic Binet's equation"

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Abstract In their recent manuscript He and Zeng claim that they have solved the general relativistic Binet's orbit equation using the exp-function method and have obtained two exact solutions useful for theoretical analysis. We argue that the obtained solutions do not satisfy the original differential equation. Moreover, we present the alternative framework for the solution of the general relativistic Binet's orbit equation.

Keywords Binet's orbit equation · Exp-function method · Solitary solution

1 Introduction

He and Zeng (2009) search for new exact solutions to the general relativistic Binet's orbit equation, which is obtained from the geodesic equation in the Schwarzschild spacetimes (Saca 2008; D'Eliseo 2007, 2009):

$$\frac{d^2u}{d\varphi^2} + u = \frac{\mu}{l^2} + 3\alpha u^2, \tag{1}$$

where $\alpha = GM/c^2 \equiv \mu/c^2$ is the gravitational radius of the central body, and c is the speed of light.

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He and Zeng (2009) assume that the solution to (1) can be expressed in the form

$$\hat{u}(\varphi) = \frac{a_1 \exp(\varphi) + a_0 + a_{-1}\exp(-\varphi)}{\exp(\varphi) + b_0 + b_{-1}\exp(-\varphi)}, \tag{2}$$

and determine constants a_1, a_0, a_{-1}, b_0 and b_{-1} by substituting (2) into (1) and by equating the coefficients of $\exp(n\varphi)$ ($n = -3; -2; -1; 0; 1; 2$ and 3) to be zero. By doing that, He and Zeng claim to use the Exp-function method (He and Wu 2006) for solving the Binet's equation and do obtain two sets of constants (He and Zeng 2009):

$$\begin{aligned}
a_0 &= \frac{1}{2B}\left(1 + \sqrt{1-4BA}\right),\\
a_1 &= a_0,\\
a_{-1} &= -\frac{4}{5}a_0^2 B/\left(1+\sqrt{1-4BA}\right),\\
b_0 &= -a_0\left(-\frac{1}{2} + \frac{1}{2}\sqrt{1-4BA}\right)\Big/A,\\
b_{-1} &= \frac{1}{(-3+2\sqrt{1-4BA})}\frac{2a_0^2}{5A^2}\left(\frac{5}{2} - \frac{5}{2}\sqrt{1-4BA}\right.\\
&\qquad \left. -9BA + 2AB\left(1+\sqrt{1-4BA}\right)\right),
\end{aligned} \tag{3}$$

and

$$\begin{aligned}
a_0 &= \frac{1}{2B}\left(1 + \sqrt{1-4BA}\right),\\
a_1 &= a_0,\\
a_{-1} &= -\frac{4}{5}a_0^2 B/\left(1+\sqrt{1-4BA}\right),\\
b_0 &= -a_0\left(-\frac{1}{2} + \frac{1}{2}\sqrt{1-4BA}\right)\Big/A,
\end{aligned} \tag{4}$$