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RESEARCH PAPER

Quantifying dispersion of finite-sized particles in deterministic lateral displacement microflow separators through Brenner's macrotransport paradigm

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Abstract Deterministic lateral displacement provides a novel and efficient technique for sorting micrometer-sized particles based on particle size. It is grounded on the principle that the paths associated with particles of different diameters, entrained in flow streaming through a periodic lattice of obstacles, are characterized by different deflection angles with respect to the average direction of the carrier flow. Theoretical approaches have been developed, which predict quantitatively the dependence of the average deflection angle on particle size. In this article, we propose an advection–diffusion model for particle transport and investigate the dispersion process about the average particle current, which controls the separation resolution. We show that the interaction between deterministic and stochastic components of particle motion can give rise to enhanced effective dispersion regimes, which may hinder separation far beyond what could be anticipated from the value of the bare particle diffusivity. The large-scale effective diffusion process is typically non-isotropic and is represented by a symmetric second-order tensor whose principal axes are not collinear with the mainstream direction of the carrier flow, or with the average particle current. The enhanced dispersion regimes can be efficiently predicted by a tailored if unconventional implementation of Brenner's macrotransport paradigm, which amounts to solving a system of two elliptic PDEs on the minimal periodicity cell of the device. The impact of macrotransport parameter on separation resolution is addressed in the concrete case of cylindrical obstacles arranged along a square lattice.

Keywords Lateral displacement · Macrotransport theory · Particle sorting · Fractionation · Effective diffusivity · Periodic lattice

List of symbols

Latin symbols

a	Particle radius (Fig. 3)
$\mathbf{e}_1, \mathbf{e}_2$	Lattice vectors (Fig. 1)
$\mathbf{r}$	Intracellular position vector (Fig. 4)
$\mathbf{u}(\mathbf{r})$	Local velocity vector of the carrier fluid
$u(x_1, x_2)$	Horizontal component of the carrier flow velocity w.r.t. x_1x_2 axes (Eq. (34))
$v(x_1, x_2)$	Vertical component of the carrier flow velocity w.r.t. x_1x_2 axes (Eq. (34))
$\mathbf{x} = (x_1, x_2)$	Global coordinate system aligned with the average carrier flow velocity $\mathbf{U}$ (Fig. 2a)
$\mathbf{x}^* = (x_1^*, x_2^*)$	Global coordinate system aligned with the lattice vector $\mathbf{e}_2$ (Fig. 2a)
$\mathbf{B}_\pi(\mathbf{r})$	Periodicized (dimensionless) corrector field (Eq. (28))
D_{ij}	Components of the effective diffusivity tensor $\mathbb{D}$ projected along the axis x_1x_2
D_{ij}^*	Components of the effective diffusivity tensor $\mathbb{D}$ projected along the axis $x_1^*x_2^*$
$\mathbf{J}(\mathbf{r})$	Dimensionless local particle current (Eq. (29))

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