**RESEARCH PAPER** 

## Wall polymer depletion effects on electrokinetic diffusioosmosis of power-law liquids in cylindrical capillaries

Pao-Wen Yang · Yi-Jui Chang · Hsin-Fu Huang

Received: 6 April 2013/Accepted: 13 November 2013 © Springer-Verlag Berlin Heidelberg 2013

**Abstract** We investigate electrokinetic diffusioosmotic flows of power-law liquids including the effects of a polymer-depleted Newtonian liquid layer near the wall boundaries in circular cylindrical capillaries. Semi-analytical solutions to the flow velocity distribution and volume flow rate are obtained for conditions involving finite double laver effects on the induced electric field of the electrokinetic diffusioosmosis. Results show that the flow behavior and responses of the electrokinetic diffusioosmotic flow depend not only on the wall zeta potential, diffusivity difference parameter, and flow behavior index, but also on the depletion layer thickness to Debye thickness ratio, the ratio of the flow consistency parameter of the power-law liquid core to the viscosity of the Newtonian depletion layer, as well as the exact numeric values of the flow consistency parameter and the Newtonian viscosity. Including the Newtonian depletion layer gives rise to wiggled-shaped zero flow rate border curves on the zeta potential versus diffusivity difference parameter map when the depletion layer thickness to Debye thickness ratio and the ratio of the flow consistency parameter of the powerlaw liquid core to the viscosity of the Newtonian depletion layer are close to one. These results are not identified in previous Newtonian or non-Newtonian electrokinetic diffusioosmosis literature and may likely open new possibilities and suggest new ideas in the analysis and design of diffusiophoretic separation and diffusioosmotic flow operations.

P.-W. Yang · Y.-J. Chang · H.-F. Huang (⊠) Department of Mechanical Engineering, National Taiwan University, No. 1, Section 4, Roosevelt Road, Taipei 10617, Taiwan e-mail: hfthuang@ntu.edu.tw **Keywords** Electrokinetic diffusioosmosis · Cylindrical capillaries · Power-law liquids · Wall polymer depletion · Rheology

## Abbreviations

EDL Electrical double layers

- OWPL Ostwald-de Waele power-law liquid
- μTAS Micro-total analysis systems

## List of symbols

с	Concentration of the ionic mass species (mol	/m <sup>3</sup> )
$-\nabla c_0$	$-\nabla c_0 = -\mathrm{d}c_0/\mathrm{d}z\overline{i_z} = 10^5\overline{i_z} \qquad (\mathrm{mol/m}^4),$	the
	concentration gradient	
D	Diffusivity of the ionic mass species $(m^2/s)$	
-		

 $E_{r,z}$  The *r*- or *z*-component of the electric field vector (V/m)

F = 96,500 (C/mol), the Faraday's constant

 $\overline{i_z}$  The unit vector in the *z*-direction

- $I_{0,1,2}$  The modified Bessel function of the first kind of order zero, one, two
- $j_z$  The z-component of the ionic mass species molar flux (mol/m<sup>2</sup> s)
- $J_{fz}$  The z-component of the volumetric-free current density (A/m<sup>2</sup>)
- *L* The characteristic length in the *z*-direction of the capillary (m)

*m* The flow consistency parameter of the power-law core flow (Pa  $s^n$ )

- *n* The flow behavior index of the power-law core flow
  - The pressure in the flow field (Pa)
  - The total volumetric flow rate (m<sup>3</sup>/s),  $Q^* = Q/\pi R^2 U_{ch}$

The *r*-coordinate,  $r^* = r/R$ 

*R* The capillary radius (m)

p

Q

r