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ORIGINAL ARTICLE

Equilibrium points and zero velocity surfaces in the restricted four-body problem with solar wind drag

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Abstract We have analyzed the motion of an infinitesimal mass in the restricted four-body problem with solar wind drag. It is assumed that the forces which govern the motion are mutual gravitational attractions of the primaries, radiation pressure force and solar wind drag. We have derived the equations of motion and found the Jacobi integral, zero velocity surfaces, and particular solutions of the system. It is found that three collinear points are real when the radiation factor $0 < \beta < 0.1$ whereas only one real point is obtained when $0.125 < \beta < 0.2$. The stability property of the system is examined with the help of Poincaré surface of section (PSS) and Lyapunov characteristic exponents (LCEs). It is found that in presence of drag forces LCE is negative for a specific initial condition, hence the corresponding trajectory is regular whereas regular islands in the PSS are expanded.

Keywords Restricted four-body problem · Radiation pressure · Zero velocity surface · P-R drag · Solar wind · PSS · LCE

1 Introduction

In space dynamics there are a number of systems like two-body, three-body, four-body, N-body problem etc. The simplicity and elusiveness of the three-body problem in different forms, like the restricted three-body problem (RTBP), restricted four-body problem (RFBP) (which may be considered as an approximation of the two three-body problem) etc. have attracted the attention of researchers for centuries. The motion of a spacecraft or satellite in the Sun–Earth–Moon system is a simple example of RFBP in space. The restricted four-body case has many possible uses in dynamical systems; for example, the fourth body is very useful for saving fuel and time in the trajectory transfers in the restricted four-body problem (Machuy et al. 2007).

The description of the effect of radiation pressure force was first time given by Poynting (1903) and the effect of the total radiation force on a particle P due to radiation source **S** was analyzed by Robertson (1937) with the help of the theory of general relativity. He stated that if we consider only the first order term in $\frac{\vec{v}}{c}$ then it consists of a justifiable approximation in classical mechanics yielding (Ragos et al. 1995)

$$F = F_{\mathrm{p}}\left(\frac{\vec{r}}{r} - \frac{\vec{v}.\vec{r}}{cr}\frac{\vec{r}}{r} - \frac{\vec{v}}{c}\right), \tag{1}$$

where $F_{\mathrm{p}} = \frac{3LM}{16\pi R^2 \rho A c}$ denotes the measure of the radiation pressure force, $\vec{r}$ is the position vector of P with respect to the Sun, $\vec{v}$ is the corresponding velocity vector, and c is the velocity of light. In expression of F_{p}, L is the luminosity of radiating body whereas M, ρ, and A are the mass, density, and cross section of the particle respectively. In Eq. (1) on the right hand side, the first term expresses the radiation pressure and the second term represents the Doppler shift owing to the motion of the particle, whereas the third term comes on account of the absorption and subsequent re-emission of the radiation. The last two terms are called the P-R drag effect.

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