

Electrophoresis of a spherical particle in a spherical cavity

Tai C. Lee · Huan J. Keh

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Abstract The electrophoretic motion of a charged spherical particle situated at an arbitrary position within a charged spherical cavity along the line connecting their centers is studied theoretically for the case of thin electric double layers. To solve the electrostatic and hydrodynamic governing equations, the general solutions are constructed using the two spherical coordinate systems based on the particle and cavity, and the boundary conditions are satisfied by a collocation technique. Numerical results for the electrophoretic velocity of the particle are presented for various values of the zeta potential ratio, radius ratio, and relative center-to-center distance between the particle and cavity. In the particular case of a concentric cavity, these results agree excellently with the available exact solution. The contributions from the electroosmotic flow occurring along the cavity wall and from the wall-corrected electrophoretic driving force to the particle velocity are equivalently important and can be superimposed due to the linearity of the problem. The normalized migration velocity of the particle decreases with increases in the particle-to-cavity radius ratio and its relative distance from the cavity center and increases with an increase in the cavity-to-particle zeta potential ratio. The boundary effects on the electrokinetic migration of the particle are significant and interesting.

Keywords Electrophoresis · Electroosmosis · Colloidal sphere · Boundary effect · Spherical pore

T. C. Lee · H. J. Keh (✉)
Department of Chemical Engineering, National Taiwan University, Taipei 10617, Taiwan, Republic of China
e-mail: huan@ntu.edu.tw

1 Introduction

A charged solid surface in contact with an electrolyte solution is surrounded by a diffuse cloud of ions carrying a total charge equal and opposite in sign to that of the solid surface. This distribution of fixed charge and adjacent diffuse ions is known as an electric double layer. When a charged colloidal particle is subjected to an external electric field, a force is exerted on both parts of the double layer. The suspended particle is attracted toward the electrode of its opposite sign, while the ions in the diffuse layer migrate in the other direction. This particle motion is termed electrophoresis and has long been applied to the particle analysis and separation in a variety of physico-chemical and biomedical systems (Hunter 1981; Masliyah and Bhattacharjee 2006).

The electrophoretic velocity U_0 of a dielectric particle of arbitrary shape and thin double layer (relative to the local radii of curvature of the particle) in an unbounded ionic solution is related to the uniformly imposed electric field E_∞ by the well-known Smoluchowski equation (Morrison 1970; Anderson 1989),

$$U_0 = \frac{\varepsilon \zeta_p}{\eta} E_\infty, \quad (1)$$

where η and ε are the viscosity and permittivity, respectively, of the fluid, and ζ_p is the zeta potential associated with the particle surface. Since the thickness of the double layer usually ranges from several to tens of nanometers, which is much smaller than the typical particle size, Eq. 1 has been used widely in practice.

On the other hand, the interaction between the ions in the mobile portion of the double layer adjoining a charged solid surface with the zeta potential ζ_w and an external electric field generates a tangential velocity for the fluid